Mathematical Modeling: Implications for Teaching

Maria Salett Biembengut & Nelson Hein
University Regional of Blumenau – BRASIL

ABSTRACT: In this paper we present the main implications of Modeling in the teaching of Mathematics where empirical data was obtained from the use of mathematical modeling for teachers through Courses of Continuing Education. The objectives of the research were to verify the possibilities and difficulties in establishing modeling as a teaching methodology. The experiment was made in four Courses given to 105 teachers. The main difficulty presented in terms of teachers' education was their lack of experience with tasks of this nature. It is rather rare for teachers' Mathematics training programs to include any orientation regarding Modeling, whether in the use of the process or its formal teaching. In spite of the difficulties, research has shown that the adoption of mathematical models in teaching can lead to better achievements for teachers and students, becoming one of the chief agents for change.

Introduction: Movements in international educational concerning mathematical modeling in teaching have also influenced Brazil (practically at the same time, starting in the 1970's), with Brazilian educators collaborating to represent the country in that part of the international academic community involved in Mathematics Education. Similarly, Mathematical Modeling, whether as a discipline in Engineering Courses and Economics, among others, or as a proposal for integrating mathematics with reality for primary and middle grade students has become one of the methodological processes used in school education.

The amount of research and reports of classroom experiences presented at Mathematics Education events (regional, statewide and national) and at the National Modeling Congress (held on a biennial basis since 1999) has increased significantly, along with the number of teachers who have become interested in the subject through extension and post-graduate courses and publications. Furthermore, various undergraduate courses for mathematics teachers have sought to include Modeling in their course curricula, either as a separate subject or as part of the overall discipline known as Teaching Methodology in Mathematics. However, despite the growing interest in modeling we see that a good deal of math teachers still do not use mathematical modeling as a classroom teaching practice. Except for some isolated cases, mathematics learning has fallen short of what had been hoped for, according to data recently revealed by the National Institute for Evaluation and Research the governmental evaluation agency responsible for Education (Biembengut, 2004).

Considering that mathematics not only greatly contributes to exercising the intellect but is also the language of science, and that mathematical modeling does follow the rules of scientific investigation, in other words, the four conditions that should be satisfied in the scientific proposition (Bassanezi, 2002; Maturana and Varela, 2001), over last decade we have made research into the development of a method that we refer to as (mathematical) Modeling Education, which uses the main principles of modeling in the teaching process and in the learning of Mathematics at every school level..

With the purpose of first identifying and understanding both the possibilities and difficulties faced by teachers when trying to make mathematical modeling a classroom teaching practice, we held four Continuing Education Courses for teachers over the course of two semesters in 2006. A total of 105 grade school teacher participated (5^{th} to 8^{th} grades). The course was made up of two parts: the first was for teachers to learn how to make a mathematical model, while the second was how to adapt the modeling process to the teaching of mathematics by following the method, which was also presented. Analysis of data obtained from this research was based on work developed by the teachers, in the observation of activities they made during the process and in interviews made at the end of the Course. In this article we present the main implications of Modeling Education in the teaching of Mathematics, based on this experimental activity. Therefore, we will first present a brief explanation of Modeling Education as a method for teaching and researching.

Modeling Education as a method of teaching and researching: Mathematical Modeling, like teaching methodology and learning, starts with a theme/subject and then develops questions about it. These questions should be answered by using mathematical tools and existing research on the theme. In a way, of course, it is highly pleasing to investigate a subject that can help students acquire a significant amount of mathematical knowledge, or to develop knowledge on the theme being studied.

Many defend the idea that each student can choose a theme/subject of any area of his/her interest, to do research on it, formulate questions and with the teacher's guidance, design a mathematical model. In this way, students become co-responsible for their learning and the teacher becomes an advisor. Learning becomes richer, considering that the student does not just learn mathematics inserted in the context of another area of knowledge, but also has his critical and creative senses stirred. In formal teaching, there are factors such as curriculum, class schedules, number of students per class and available for the teacher to follow up the students' works. Because of these variables we had to adapt somewhat the process of mathematical modeling to teaching methodology and learning, establishing a method that that we call Modeling Education (Biembengut, 1997; Biembengut, 2004).

Modeling Education is a teaching and research method that uses the essence of Modeling. Research has shown that the application of the method is expected to provide the student with: integration of mathematics with other areas of knowledge; interest in the application of mathematics; improvement in grasping mathematical concepts; incentive for creativity in the formulation and resolution of problems; ability to use of machines (graphic calculators and computers); capacity to act in a group; orientation in doing research and capacity for reporting research (Biembengut and Hein, 2007). Modeling Education is guided by the teaching of the program content, starting from applied mathematical models and then moving into several areas of knowledge; at the same time guiding students in the direction of research. It can be implemented at any educational level, from Elementary School Teaching to the University.

Modeling Education as a teaching method: Modeling Education as a method of teaching mathematics aims at providing the student with a better apprehension of mathematical concepts; training him/her to read, to interpret, to formulate and to solve specific situation-problems, as

well as to awaken his critical and creative senses. To apply it in teaching, the teacher chooses a theme/subject of any area of knowledge that can be of interest to students (according to the content of the program), and designs a mathematical model, adapting it to teaching. Alternatively, he/she chooses an existing mathematical model and adapts it to the development of the program content. That model will then serve as a modeler. This involves the teacher in a series of activities/stages that consist in (Biembengut and Hein, 2005):

1st. Exhibition of the theme: The teacher begins the class by giving a brief explanation of the theme/subject to the students, urging them to raise questions about the chosen theme;

2nd. Delimitation of the problem: The teacher selects one or more questions which allow him to develop the program content. If it is possible and/or convenient, he can propose to the students that they do research on the subject, either a bibliographical study or an interview with some expert in the subject area.

3rd. Formulation of the problem: The teacher starts to formulate the problem by showing hypotheses, calculating or organizing the data in such a way that it asks for mathematical content for its resolution.

4th. Development of the program content: Here the program content is presented (concept, definition, properties etc.) linking it with the subject that generated the process.

5th. Presentation of similar examples: Immediately after these steps, similar examples are presented, enlarging the options for application, and thus avoiding any chance that the content becomes limited to the theme or presented subject. Stimulus and guidance in the use of technological devices like calculators and/or computers that are part of classroom practice is also important.

6th. Formulation of a mathematical model and solution of the problem departing from the model: The teacher proposes to the students that they come back to the problem that generated the process and solve it.

7th. Interpretation of the solution and validation of the model: Ending at this stage, it is important that the student evaluates the result – validation. This allows the student to develop a better understanding or comprehension of the obtained results.

Modeling Education as a research method: The central objective of this work is to create conditions for students to learn how to do research and designing applied mathematical models from some other area of knowledge. This work is accomplished parallel to the development of the program content. We have suggested that students group themselves according to their interests and personal preferences and that the school period be divided into at least five stages, so that the proposals can be accomplished and the teacher can properly guide the class. The stages are:

1st . Theme choice: Groups are formed with a maximum of 4 students, and each group chooses a theme/subject according to individual interests. The groups, with the teacher's guidance, should

be responsible for the choice and direction of their own work. When the theme/subject is chosen the teacher proposes that data be collected through special bibliographies or/and by interviewing experts in the area,

2nd . Becoming familiar with the theme to be modeled: In this second stage students should already be familiarized with the theme and have much data at their disposal. The teacher then proposes that a series of questions be raised and that they elaborate a synthesis of the research to be given to him. That synthesis allows the teacher to learn about the theme and to select, as a suggestion, about 3 questions to be addressed by each group. These questions should include mathematical notions which are part of the program.

3rd . Delimitation of the problem and formulation: After having delimited the problem or selected the questions to be answered, the teacher starts to formulate the problem from the questions that ask for more elementary mathematical concepts for their solution. When the group has a good basis about the theme that they are working on, an interview with a Specialist/Expert can greatly contribute to the work.

4th . Design of a mathematical model, solution and validation: Once the problem has been formulated, the group tries to elaborate a model that not only allows for a solution of the specific questions at stake, but that can also be applied to other solutions or that can be used to predict certain results.

5th . Organization of the written work and oral report: It is essential that the work be published. Therefore, in this stage, the groups should present the developed work developed in writing and orally, in a seminar, to the other students or to those for whom it may be of interest.

Modeling Education can only be used as a teaching method or as a research method. When using it only as a teaching method, it is suggested that the teacher guides the students' research by using a specialized bibliography and/or by interviewing a specialist/expert about the theme of the principal model(s). When using Modeling Education only as a research method, development of the program content can be given in the traditional form. Using just one of the approaches or both - teaching and research – the premise is to promote mathematical knowledge and the ability to apply it in other areas of knowledge, in other words, the objective is to supply students with elements which allow them to develop their potential, providing them with the capacity to think critically and independently.

Important Occurrences in Modeling Education: Starting from this premise, we have been applying, directly or indirectly, Modeling Education for less than two decades with teachers sympathetic to the method at the Elementary, Secondary School and University levels. We have also been developing projects with teachers through Continuing Education Course and disciplines taught in Specialization Course in Mathematical Education. Despite a number of Continuing Education Courses that we have held and the interest shown by the majority of teachers who participated, we do not have information available about the possible adoption of Modeling in their classroom practices, nor any data concerning how modeling was understood

and/or adopted by teachers that allows us to know where they are at the moment and what roots or influences lay in the past and the roads that need to be followed for the future.

With this in mind, we conducted 4 Continuing Education Courses for 4 groups, making a total of 105 teachers who agreed to participate and contribute to the research over the course of two semesters in 2006, comprising 90 hours in all. Each course followed the orientation as the method for teaching and research. As a teaching method we used 5 mathematical models and remade each one (following the routine described in Modeling Education as a teaching method above) in order for the teachers to be able to become aware of the art of mathematical modeling; to clarify mathematics content and verify the possibility of adapting the example in their classroom practices. As a research method (following the routine described in Modeling Education as a research method above), teachers formed groups of 4 people, with each group agreeing on and selecting a theme that interested them and developing a mathematical modeling project that could also be used in their classroom teaching practices. Based on observations made during Course activities, in the work made by participants and in interviews made at the conclusion of the Course, some essential results were obtained that allow us to make a re-structuring of the teacher training process. The following are the main advantages and difficulties presented by the teachers who participated in the course and which could have implications in their classroom teaching practices.

Main advantages

a) In relation to the mathematical models used to orient during the process:

- Enabled teachers to gain a better understanding of the developed content due to the approach to the target area and its application;

- Offers the teacher a safer route in the implementation of classroom activities, greatly facilitating the way classes are conducted, and also facilitating inter-relationships with other disciplines, and then to come back to the orientating model, solving it and evaluating it.

b) In relation to the modeling work:

- Leads the teachers to act/do and not just to receive ready concepts/materials without understanding the meaning of what he has been studying; to do research, not a routine activity in the classroom, in spite of being part of the curriculum; to imprint knowledge, creativity and critical sense, mainly in the formulation and validation of the model; to interact and to learn about the work of the other groups in the seminar and, among other things, to apply the norms of the scientific methodology when elaborating a written report of the work.

- Allows the teacher to be more attentive to students' difficulties; to become aware of their work in a gradual way, especially when guiding the students, and allow them to change their criteria and evaluation instruments.

Main difficulties

The main difficulty centers on teachers' training/education and their lack of experience with tasks of this nature when they were students themselves. It is rather rare for teachers' Mathematics training programs to include any orientation on Modeling, whether in the use of the process or its formal teaching. The inclusion of this content has been more frequent in the last decade in Extension Courses of Extension or in disciplines taught in Masters' Degrees Programs in Mathematics Education. Among the difficulties presented by the teachers who participated in the Course we identified two types: one during the process of designing a mathematical modeling project and the other gleaned from the interview given at the end of the course.

a) Difficulties experienced by teachers in the stage of Modeling Education as a research method:

– Interpretation of the context: In traditional teaching, especially in Mathematics, learners are rarely presented with situations or problems that ask for a reading and an interpretation of their context and then to proceed to the formulation and explanation of that context. Without that living experience, a student or professional will lose this capacity. Any attempt of rescuing it is not an easy task. Traditional teaching doesn't make the student qualified to read the context, reading, in a wide sense, the word. Abilities are rarely developed to enable the reading of music, a piece of art, poetry, a historical context, a political situation, a statistical result, among others. This is one of the most important flaws of current education. In these terms, whenever these teachers were faced with a text or context, they experienced serious difficulties in reading, understanding, and interpreting, that is, in making a reading.

– Availability for research: The themes chosen demand research, research which has to be done outside the Course. However, since the majority of teachers did not have any time available for out-of-class orientation activities this was not always possible. When discussing work in modeling, it is worth pointing out that the time that students have available for collecting data and studying, has an important effect on developing quality work.

– Choices of the theme/subject: The choice of a theme was not an easy task. Each group of teacher chose a subject of interest, but did not always achieve expected results. For instance, some groups chose themes whose data didn't bring any additional knowledge to what concerns mathematics, and others chose themes whose data was not easy to obtain. In this case, in order to avoid problems in the middle of the process we have to encourage participants to look for other topics.

– Work in Group: A theme chosen by the group didn't assure that all, in fact, were interested in it. This required a deeper involvement from each participant in order to accomplish what was proposed. This lack of involvement, along with the lack of commitment by some teachers compromised the work of the group as the essence of this work is cooperation and teamwork in the construction of knowledge.

b) Difficulties presented by the teachers in interviews regarding the implementation of Modeling Education as a teaching method and research method.

- Improvement: Since Modeling and Modeling Education have been defended as teaching methods for less than three decades and considering the geographical dimensions of our country, improvement/continuing education courses and specialization courses in this subject have not yet reached all educators in the area. Limited to 90 hours, this Course was not enough for complete training in the method. It was just enough to 'signal' the subject, provoking a certain motivation on the part of the teachers.

- Bibliography: There are few studies in Portuguese about the use of Modeling and Modeling Education in teaching that have been published or that are available or easily accessible to teachers. Despite the fact that most areas of knowledge apply mathematical models (Physics, Chemistry, Biology, Economics, etc.), teachers state that they do not have sufficient knowledge in these areas to be able to interpret these mathematical models in the scarce amount of time available to them.

- Guidance: A single course or text about this subject does not provide sufficient background for the teacher to immediately put the method of Modeling Education into practice. Security and ability are acquired with time. Professional orientation in the subject - explaining difficulties and aiding in the planning and organization of the activities would provide a safer ground for the teacher. On way could be supervision by virtual means; in this case, we would require researchers to plan activities that include this kind of guidance.

- Planning: Planning means to establish the strategies that should be used in explaining and referring problems to the learning process, to the structure and form adopted and to the practices applied towards better evaluation of the process and results. However, for the majority of participating teachers, planning is included in the list of contents and in forms of evaluation that they will adopt over the course of the school year; later, they will adopt a textbook to follow. In implanting mathematical modeling in teaching, the teacher will need to employ careful planning in deciding when to teach a particular topic of the program and to present similar examples, all integrated into the students' work. For example, in order to be able to guide students in their work, the teacher will have to know something of the theme/subject. The larger the number of students' groups and the larger the number of themes, the longer the time the teacher will need to study the projects. This demands that the teacher have enough time available to do the task, which the teacher does not have.

Final considerations: We understand that the objective of teaching, at all levels, should be to provide students with opportunities to acquire knowledge and to develop attitudes and abilities that help them to fully interact with society. It is with this objective that we have been defending Modeling Education as a teaching and research method. Modeling Education, however, is not a panacea to overcome all problems of school practice in the teaching of Mathematics. Research points out that it can represent progress in the teaching of Mathematics in the classroom because when it is used Mathematics stops being a mere transmission of resolution techniques and becomes a tool or structure in another area of knowledge. This demands a deeper commitment to studies, to research and to interpretation of context, for teachers as well as for the students. In other words, it means much more work!

As researchers, we hope to implement effective teaching at all levels of education because there is an imperative need for continuous improvement, especially for teachers who work in training and education programs. It is not enough to teach mathematics content but necessary to also make a commitment to the training of these future teachers. We are not at work in an area simply to explore it. If basic education is precarious, it is because teacher education courses are also precarious.

The research made here, in spite of the difficulties, shows that the adoption of mathematical models in teaching, whether in presentation form or in the creation process, when appropriately adapted to the realities of the school, is a means of providing teachers and students with better chances of success, becoming one of the chief agents for change. With this in mind, the preliminary challenges for researchers stem from recognizing the problems that emerge from our precarious professional training and by confronting the changes inherent to such a situation. “There are no mysteries in the basic elements involved in the process for us to reach that condition; all the necessary technologies, tools and elements of changes exist. The real challenge is to decide to be committed to a new way of action. This new perspective needs to be embraced by each of us. (Brown, 1991).

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